Mathematics (for Physics) for Biologists

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OIST

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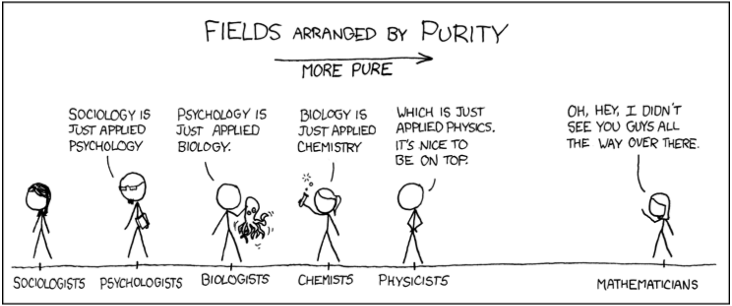


R. Munroe, *Purity*, web comic, June 2008

Thinking about numbers

- How many piano tuners are there in New York City?
- Is it true that there are more connections in the human brain than atoms in the universe?
- Why did deaths from iron poisoning in small children in the US drop dramatically after 1998?
- Is it true that we use only 10 % of our brain?
- How much does it cost to set up a *C. elegans* lab and run it for a year?
- How many different coffees can you have at Starbucks?

Looking at equations

Michaelis-Menten kinetics

$$v = \frac{V_{\text{max}}[S]}{K_{\mathrm{M}} + [S]}$$

v	...	reaction rate
v_{max}	...	maximal rate
$[S]$	...	substrate concentration
K_{M}	...	Michaelis constant

Questions to ask when looking at equations

Questions to ask when looking at equations

- What do the terms mean? Which of them are constant, which ones are variable?
- How does the whole change when your variable of interest changes? What is the shape of the curve?
- What are the extremes? Can this ever be less than zero/zero/infinitely big/not defined?
- What happens in "special cases" (e.g. your variable of interest is zero, infinity, . . .)?
- What can I measure?
- Why do I care?

Looking at equations

Drake equation:

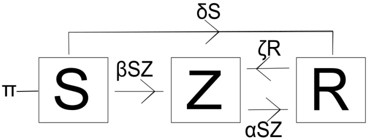
$$N = R_* \cdot f_p \cdot n_e \cdot f_\ell \cdot f_i \cdot f_c \cdot L$$

N	...	number of civilizations in our galaxy with which radio-communication might be possible
R*	...	average rate of star formation in our galaxy
fp	...	fraction of those stars that have planets
ne	...	average number of planets that can potentially support life per star that has planets
fl	...	fraction of planets that could support life that actually develop life at some point
fi	...	fraction of planets with life that actually go on to develop intelligent life (civilizations)
fc	...	fraction of civilizations that develop a technology that releases detectable signs of their existence into space
L	...	length of time for which such civilizations release detectable signals into space

Looking at equations

Munz model:

$$S' = \Pi - \beta SZ - \delta S$$
$$Z' = \beta SZ + \zeta R - \alpha SZ$$
$$R' = \delta S + \alpha SZ - \zeta R$$



P. Munz *et al.*, *In: J.M. Tchuenche and C. Chiyaka, editors, Infectious Disease Modelling Research Progress* (Nova Science, 2009), pp. 133–150

Choose your own adventure

- Thinking about numbers ▸ Go
- Thinking about equations ▸ Go
- Differential and integral calculus ▸ Go
- Differential equations ▸ Go
- Matrix operations ▸ Go
- Combinatorics ▸ Go

Differential calculus

$$\frac{d}{dx}(x^2 + 3x + 4)$$

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Differential calculus

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$$\frac{d}{dx}x^2\cos(x)$$

$$\frac{d}{dx}e^{x^2}$$

Integral Calculus

$$\int (x^2 + 3x + 4)dx$$

Integral Calculus

$$\int (x^2 + 3x + 4) dx$$

$$\int e^x dx$$

Integral Calculus

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$$\int \frac{1}{x} dx$$

Integral Calculus

$$\int (x^2 + 3x + 4) dx$$

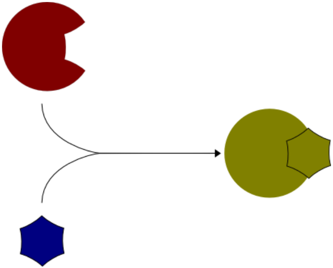
$$\int e^x dx$$

$$\int \frac{1}{x} dx$$

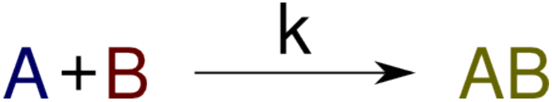
$$\int (\sin x)(\cos x) dx$$

On to differential equations
Back to overview

Differential equations



Differential equations



Chemical reactions

$$\frac{d[A]}{dt} = -k[A][B]$$

$$\frac{d[B]}{dt} = \frac{d[A]}{dt} = -k[A][B]$$

$$\frac{d[AB]}{dt} = k[A][B]$$

Chemical reactions

$$\frac{d[A]}{dt} = -k[A][B]$$

$$\int \frac{1}{[A]} d[A] = -\int k[B]dt$$

$$\log[A] = -k[B]t + c_0$$

$$[A] = c_1 e^{-k[B]t}$$

At $t = 0, [A] = [A]_0$ and $e^{-k[B]t} = 1$. Hence:

$$[A] = [A]_0 e^{-k[B]t}$$

What's this model?

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$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \delta xy - \gamma y$$

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$$\frac{dx}{dt} = \alpha x - \beta xy$$

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Lotka-Volterra-Model

Now that we have talked about ODEs . . .

- Don't specify your ODEs by hand.

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- Your computer probably won't.

Now that we have talked about ODEs . . .

- Don't specify your ODEs by hand. *It's tedious and error-prone.*
- Don't solve your ODEs yourself. *Your computer will do it.*
- Your computer probably won't. *It will use numerical approximations.*

Euler's method

- Start at initial condition.
- Compute the slope there.
- Follow the slope for "a little bit".
- Compute the slope again.
- Follow the slope for "a little bit" again.
- And so on . . .

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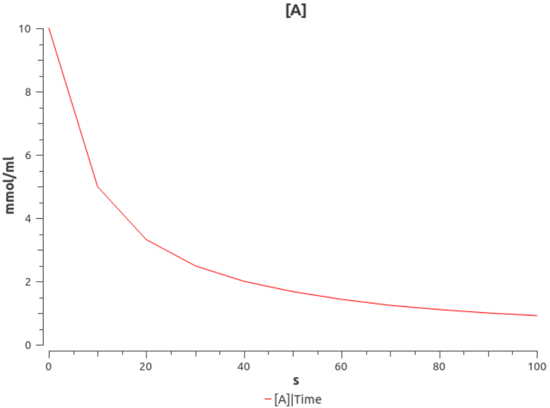
- And so on ...

$$\frac{d[A]}{dt} = -k[A][B]$$

Let's say $[A]_0 = [B]_0 = 10\,\mu\text{M}$, $k = 0.01$.

Euler's method

Copasi Demo



S. Hoops *et al.*, *Bioinformatics* **22**, 3067–3074 (Dec. 2006)

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Matrix operations

Addition:

$$\begin{pmatrix} 1 & 5 \\ 7 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ -4 & 3 \end{pmatrix} =$$

Scalar product:

$$5 \times \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & -1 \\ 4 & 0.2 & 1 \end{pmatrix} =$$

Matrix operations

Dot product:

$$\begin{pmatrix} 2 & 1 & 0 \\ -2 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & -2 \\ -1 & 2 & -1 \end{pmatrix} =$$

Matrix operations

Dot product:

$$\begin{pmatrix} 2 & 1 & 0 \\ -2 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & -2 \\ -1 & 2 & -1 \end{pmatrix}$$

Matrix operations

Matrix product:

$$\begin{pmatrix} -2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 3 & -2 \\ -1 & 2 & -1 \\ 1 & 2 & 1 \\ 3 & 0 & 1 \end{pmatrix}$$

Matrix operations

Matrix product:

$$\begin{pmatrix} -2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 3 & -2 \\ -1 & 2 & -1 \\ -1 & 2 & 1 \\ 3 & 0 & 1 \end{pmatrix}$$

Leslie model

Population ecology:
Suppose a certain animal has a maximum life span of three years. The life cycle can be divided into three phases: Year 1 (0-1 yr), Year 2 (1-2 yr), and Year 3 (2-3 yr). We only consider females. A Year 1 female animal has no offspring; a Year 2 female has 3 daughters on the average; and a Year 3 female has an average of 2 daughters. A Year 1 animal has a 0.3 probability of living to Year 2. A Year 2 animal has a 0.4 probability of living to Year 3. Suppose at one instance, the number of Year 1, 2, and 3 females are 2030, 652, and 287, respectively.
What is the expected number of females in each category

- a year later?
- two years later?

A. B. Shiflet, G. W. Shiflet, *Journal of Computational Science Education* (2011)

Leslie Matrix

$$\begin{pmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{pmatrix}_{t+1} = \begin{pmatrix} f_0 & f_1 & f_2 & \dots & f_{\omega-2} & f_{\omega-1} \\ s_0 & 0 & 0 & \dots & 0 & 0 \\ 0 & s_1 & 0 & \dots & 0 & 0 \\ 0 & 0 & s_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{\omega-2} & 0 \end{pmatrix} \begin{pmatrix} n_0 \\ n_1 \\ \vdots \\ n_{\omega-1} \end{pmatrix}_t$$

n_i ... individuals in age class i
f_i ... number of female offspring per individual in age class i
s_i ... chances of surviving to the next age classs

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Dot product: Image filtering

- Decide on a filter matrix
- For each pixel in the image:
    - Take a sub-matrix of the image centered around that pixel (of the same size as the filter matrix)
    - Compute the dot product of that matrix and the filter matrix
    - Replace the value at that pixel by that dot product

Dot product: Image filtering

0	0	0	0	0	0
0	1	1	1	1	0
0	1	0	0	0	0
0	1	0	0	0	0
0	1	0	0	0	0
0	0	0	0	0	0

1	0	0
1	0	0
1	0	0

Combinatorics

- How many of your ancestors lived 1000 years ago?

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- Fragile X syndrome causes loss of function of the X-chromosomal FMRP protein. Around 1 : 4000 males are affected. How many females would you expect to be affected?

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- Fragile X syndrome causes loss of function of the X-chromosomal FMRP protein. Around 1 : 4000 males are affected. How many females would you expect to be affected?
- CaMKII exists as a dodecamer, where every subunit has 2 possible conformational states, 2 phosphorylation sites, and 2 calmodulin binidng sites. How many different forms of CaMKII are theoretically possible? How many exist in a dendritic spine at any given time?

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References I

R. Munroe, *Purity*, web comic, June 2008, https://xkcd.com/688/.

P. Munz, I. Hudea, J. Imad, R. J. Smith, *In: J.M. Tchuenche and C. Chiyaka, editors, Infectious Disease Modelling Research Progress* (Nova Science, 2009), pp. 133–150.

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